High-speed oblique drop impact on thin liquid films

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We numerically investigate high-speed drop impact on thin liquid films with a focus on oblique impact. The flow behavior is described by solving the incompressible Navier-Stokes equations using the variable density pressure projection method. The phase interfaces are captured using the moment-of-fluid method. The numerical method is validated against experiments and theoretical predictions. Our study on high-speed oblique impact reveals that the tangential velocity can significantly alter impact phenomena: a higher tangential velocity leads to a lower lamella height and radius on the side behind the advancing drop, and the higher tangential velocity also leads to stronger vortices at the drop and film interface due to Kelvin-Helmholtz instability. Our investigation on the effect of liquid film thickness shows that a thinner liquid film leads to an earlier crown breakup. Last, our study shows that lowering the film density can prompt earlier splashing. Published by AIP Publishing.

I. INTRODUCTION

The phenomenon of drop impingement on a wall has attracted much attention due to its various practical applications such as spray cooling, inkjet printing, and pesticide spraying. Although low-speed normal impacts are involved in most of these applications, high-speed oblique drop impact is critical for the studies of aircraft icing, fuel injection in internal combustion engines, steam turbines, etc. Many experimental and numerical studies\(^1\)–\(^3\)\(^1\)–\(^3\) have been conducted to understand the drop impact phenomenon. These studies revealed that the morphologies and underlying mechanisms of dry surface impact and wet surface impact are fundamentally different.\(^2\),\(^3\)

When a drop impacts on a dry surface, the drop can deposit, rebound, or splash, depending on parameters such as drop size, impact velocity, surface tension, substrate properties, and surrounding gas density.\(^1\),\(^2\),\(^4\) Mundo et al.\(^5\) experimentally showed that there are two possible outcomes from the impingement of drops on dry solid surfaces: less energetic drops deposit on the surface and form a liquid film, while energetic drops splash and form secondary drops. The authors characterized the transition from deposition to splashing using the parameter \(K = Re^{0.25} \cdot We^{0.5}\), where the splashing threshold is 57.7. The Reynolds number and the Weber number are defined as

\[
Re = \frac{\rho D_0 V_0}{\mu}, \quad We = \frac{\rho D_0 V_0^2}{\sigma},
\]

where \(\rho\), \(\mu\), and \(\sigma\) are the liquid density, viscosity, and surface tension, respectively, and \(D_0\) and \(V_0\) are the drop diameter and impact velocity, respectively. Experimental studies by Xu et al.\(^6\) and Latka et al.\(^7\) found that air plays a crucial role in determining the dry surface splashing behavior as low pressure can effectively suppress the splashing. Guo et al.\(^4\) numerically investigated the role of ambient air in drop splashing on dry and wet surfaces, confirming that the splashing can be suppressed when the ambient air effects are negligible and the thin film splashing is not affected by the ambient air prominently.

Under most practical circumstances, drop impact on dry surfaces is less common. Instead, surface is often wet from previous drop impingements, and hence drop often impacts on surface covered by a liquid layer. When a drop impacts on a liquid layer or wet surface, splashing is generally observed and coalescence occurs only when the drop momentum is low.\(^1\),\(^2\)

The liquid layer can be described as a film or pool depending on the non-dimensional parameter \(\delta\), defined as \(h_0/D_0\), where \(h_0\) is the liquid layer thickness and \(D_0\) is the drop diameter. When \(\delta\) is less than 1, it is referred to as a liquid film.\(^8\),\(^9\)

Experimental studies of thin liquid film impact\(^28\)–\(^30\) classified different regimes of motion after impact, depending on the Reynolds number, the Weber number, and the ratio of film thickness to drop diameter. For very low impact velocity, the drop deposits onto the thin liquid film without crown formation. For high impact velocity, spreading occurs if a radially expanding crown is observed but does not break up into smaller secondary drops. When the \(K\) number is greater than a threshold, splashing occurs through a two-stage process: in the first stage, a horn-shaped jet is formed from the neck region between the drop and liquid layer, and the jet develops to form a crown shape; in the second stage, the crown tip breaks up to secondary drops due to instability.

Various models based on experiments have been proposed to predict the splash threshold for drop impact on the liquid layer. Table I lists several commonly used models.\(^8\),\(^20\),\(^21\),\(^29\)

Figure 1 compares these splash threshold models for water drop impact on thin films of \(\delta = 0.1\). The model of Yarin and Weiss\(^20\) predicts the highest threshold impact velocity for splashing among all and the difference in the predicted threshold velocities is larger for smaller drops.

Other studies have been conducted to investigate the jet formation and the secondary drop formation. Weiss and...
Yarin simulated drop impact on a liquid film to study the formation of a jet in the neck region between the drop and liquid film after impact. For sufficiently energetic impacts, the authors found that the jet can pinch off a torus-shaped liquid volume at its tip or reconnect with the liquid film. Thoroddsen and Richtmyer-Meshkov studies, experiments by Ching et al., and Zhabnkova and Kolpakov experimentally verified jet formation in the neck region between the drop and liquid film. The mechanism of secondary drop formation was also investigated. Different mechanisms were used to explain the crown formation instability in splashing, including the Rayleigh-Taylor instability, Rayleigh capillary instability, and Plateau-Rayleigh capillary instability, and Richtmyer-Meshkov instability.

Most previous studies focused on normal impact with relatively low impact velocity (<30 m/s) and film thickness, δ, greater than 0.1. Study of high-speed oblique impact on a thin liquid film is rare. Among the few oblique drop impact studies, experiments by Ching et al., Leneweit et al., and Zhabnkova and Kolpakov investigared oblique impact on a liquid pool (δ > 1) in the non-splash regime. Okawa et al. experimentally studied the effect of impact angle on the total mass of secondary drops produced during the collision. Liang et al. experimentally studied the effects of surface tension and viscosity on spreading and splashing behaviors. Cheng and Lou numerically studied drop oblique impact (30° ≤ θ ≤ 90°) on a wet wall using the lattice-Boltzmann model and observed a transition from crown splashing to single-sided splashing; however, their model artificially introduced a cosine-wave perturbation on the surface of a drop before impact to form splashing.

In the present study, we numerically investigate the high-speed drop impacts on thin liquid films. The paper is structured as follows. First, we present the numerical method. Second, we validate our code with experiment and theoretical solution. Subsequently, we investigate normal and oblique impacts of water drops on water films of different impact angles and film thicknesses. Finally, we study the normal impacts of different film-drop density ratios.

II. NUMERICAL METHOD

The numerical method was discussed in detail in our earlier papers. Here we briefly present the key steps with an emphasis on interface reconstruction when more than two materials exist.

A. Governing equations

The governing equations for incompressible, immiscible, multiphase flows are

\[ \nabla \cdot \mathbf{U} = 0, \]

\[ \frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \otimes \mathbf{U}) = -\nabla p + \nabla \cdot (2\mu \mathbf{D}) + \rho \mathbf{g} - \sum_{m=1}^{M} \gamma_{m} \kappa_{m} \nabla H(\phi_{m}). \]

\[ \frac{\partial \phi_{m}}{\partial t} + \mathbf{U} \cdot \nabla \phi_{m} = 0 \quad m = 1, \ldots, M, \]

where \( \mathbf{U} = (u,v,w) \) is the velocity vector, \( t \) is the time, \( p \) is the pressure for material, \( \mathbf{g} \) is the gravitational acceleration vector, and \( D \) is the rate of deformation tensor,

\[ D = \frac{\nabla \mathbf{U} + (\nabla \mathbf{U})^{T}}{2}, \]

where \( \mu = \sum_{m=1}^{M} \mu_{m} H(\phi_{m}) \) is the viscosity, \( H \) is the Heaviside function defined as

\[ H(\phi) = \begin{cases} 1 & \phi \geq 0 \\ 0 & \text{otherwise} \end{cases}. \]

The stress at the material interface will have the following jump condition:

\[ \left[ (-pI + 2\mu \mathbf{D}) \cdot \mathbf{n} \right] = \sigma \kappa \mathbf{n}, \]

where \( \sigma \) is the surface tension coefficient and \( \kappa \) and \( \mathbf{n} \) are the curvature and unit normal of the interface, respectively. The interface unit normal is

\[ \mathbf{n} = \frac{\nabla H}{|\nabla H|}, \]

\[ \phi_{m} \] is the level set function for material \( m \) and satisfies

\[ \phi_{m}(\mathbf{x}, t) = \begin{cases} > 0 & \mathbf{x} \in \text{material } m \\ \leq 0 & \text{otherwise} \end{cases}. \]

B. Moment-of-fluid interface reconstruction

The moment-of-fluid (MOF) method is used to reconstruct interfaces between different phases. The MOF
method, which can be considered as a generalization of the volume of fluid (VOF) method, uses both the volume fraction function and the corresponding material centroid to construct the interface. Unlike the VOF method, the MOF interface reconstruction method only uses information from the computational cell under consideration. This characteristic makes the MOF method more suitable for block structured adaptive mesh refinement.

In the MOF method, we represent the material \( m \) distribution using its volume fraction and centroid. In each computational cell \( \Omega_{i,j,k} \), the volume fraction of material \( m \) is the zeroth-order moment,

\[
F_m = \frac{1}{\Omega_{i,j,k}} \int_{\Omega_{i,j,k}} H(\phi_m(x))dx, \tag{10}
\]

and the centroid of the material is the first-order moment,

\[
x_m = \frac{\int_{\Omega_{i,j,k}} H(\phi_m(x))xdx}{\int_{\Omega_{i,j,k}} H(\phi_m(x))dx}. \tag{11}
\]

In each computational cell, the interface between different materials is represented by a plane in a three-dimensional grid or a line in a two-dimensional grid, which is called the piecewise linear interface calculation (PLIC). For a two-dimensional grid, the interface is represented by a straight line as shown in Fig. 2 using the following equation:

\[
n \cdot (x - x_{i,j}) + b = 0, \tag{12}
\]

where \( n \) is the unit normal vector, \( x_{i,j} \) is the cell center, and \( b \) is the distance from the cell center to the line.

When there are only two materials in a computational cell, the interface can be reconstructed using the volume fraction and centroid of any material. As shown in Fig. 3, red and blue represent two materials. Given a reference volume fraction function, \( F_{ref,m} \), and a reference centroid, \( \mathbf{x}^c_{ref,m} \), the MOF interface reconstruction requires that the actual volume fraction function matches the reference volume fraction function exactly and the actual centroid is close to the reference centroid as illustrated in Fig. 3. This procedure can be achieved by the following constraint optimization problem:

\[
E_{MOF} = \left\| \mathbf{x}^c_{act,m} (\mathbf{n}, b) - \mathbf{x}_{act,m} \right\|^2
\]

\[
s.t. \quad \left| F_{act,m} (\mathbf{n}, b) - F_{ref,m} \right| = 0. \tag{13}
\]

The step-by-step procedure is described by Jemison et al.\(^{16} \)

In our numerical models, we used two different materials to differentiate the liquid drop and the liquid film to illustrate the pertinent fluid dynamics features in the drop impact on a thin film. When there are more than two materials in a computational cell (i.e., liquid drop, liquid film, and air), an extension of the two-material MOF method discussed in Fig. 3 is used to reconstruct the interface. The method is illustrated in Fig. 4. We first reconstruct the interface for the material whose centroid is furthest from the centroid of the computational cell by solving Eq. (13). In the case shown in Fig. 4, it is the red material. Next, we reconstruct the interface for the remaining material whose centroid is furthest from the centroid of the unoccupied region in the cell (white region) by solving Eq. (13). In Fig. 4, the next material is marked in blue. This procedure continues until all the material interfaces are constructed.
C. Adaptive mesh refinement

The Navier-Stokes equations are solved using the variable density pressure projection algorithm\textsuperscript{47} on the block structured adaptive mesh refinement grids.\textsuperscript{48,49} The grid adaption is based on the triple point region and the curvature of the interface. As shown in Fig. 5, the grid refinement is performed near regions where curvature is greater than a predefined value. The adaptive mesh refinement method ensures that a fine grid is only used in the regions of interests, which maintains the accuracy of the solver at a reasonable computational cost.

III. RESULTS AND DISCUSSIONS

We first validate the code by comparing with experimental results and theoretical predictions using three cases. Then we investigate the effects of impact angle and film thickness by simulating the oblique impacts of 41.3-µm-diameter water drops on water films of different thicknesses at different impact angles. The film thicknesses used in the simulations are 1 µm, 5 µm, and 20 µm, and the impact angles are 60°, 45°, 30°, and 15°. Finally, we present the simulation results of the impacts of different film-drop density ratios. Table II summarizes the parameters for all simulation cases.

A schematic of the simulation setup of a liquid drop impacting onto a thin liquid film is shown in Fig. 6. The liquid drop diameter is \(D_0\) and the liquid film thickness is \(h_0\). The drop is surrounded by air above the liquid film. The impact angle is \(\theta\) and the impact velocity is \(V_0\), and \(V_{0t}\) and \(V_{0n}\) are the tangential and normal components of impact velocity, respectively. The dimensionless time is defined as \(T = \frac{V_{0n} \cdot t}{D_0}\), and the dimensionless film thickness is defined as \(\delta = \frac{h_0}{D_0}\). Due to symmetry of the problem, the computational domain of a three-dimensional model only contains one half of the drop for oblique impact and one quarter of the drop for normal impact. Although the three-dimensional model can capture ejected secondary drops and crown breakup, these phenomena are not the main focus of this study. Therefore, a two-dimensional numerical model is also used to investigate flow behaviors with higher grid resolution. The tangential velocity component points to the right in Fig. 6 and we define the right side of the drop as the upstream side and the left side as the downstream side.

A. Code validation

Three code validation study cases are performed and presented in this section. The three cases represent low-speed impact on thick liquid film, low-speed impact on thin liquid film, and high-speed impact on thin liquid film, respectively.

<table>
<thead>
<tr>
<th>Drop diameter (D_0)</th>
<th>Liquid type</th>
<th>Impact velocity (V_0) (m/s)</th>
<th>Impact angle (\theta)</th>
<th>Film thickness (h_0)</th>
<th>(\delta = h_0/D_0)</th>
<th>(We)</th>
<th>(Re)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 3.82 mm Water</td>
<td>3.94</td>
<td>90° (normal)</td>
<td>2.57 mm</td>
<td>0.67</td>
<td>842</td>
<td>15 366</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.51</td>
<td></td>
<td></td>
<td></td>
<td>667</td>
<td>13 676</td>
<td></td>
</tr>
<tr>
<td>A. Code validation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2 2 mm Silicone oil</td>
<td>1.83</td>
<td>90° (normal)</td>
<td>0.4 mm</td>
<td>0.2</td>
<td>324</td>
<td>2 191</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 3 41.3 µm Water</td>
<td>53</td>
<td>90° (normal)</td>
<td>1 µm</td>
<td>0.024</td>
<td>1 590</td>
<td>1 680</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5 µm</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Effect of impact angle 41.3 µm Water</td>
<td>53</td>
<td>30.6</td>
<td>60°</td>
<td>2121</td>
<td>1940</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>91.8</td>
<td>5 µm</td>
<td>6 362</td>
<td>3 361</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>197.8</td>
<td>15°</td>
<td>23 742</td>
<td>6 493</td>
<td></td>
</tr>
<tr>
<td>C. Effect of film thickness 41.3 µm Water</td>
<td>53</td>
<td>53</td>
<td>45°</td>
<td>3 185</td>
<td>2 378</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5 µm</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20 µm</td>
<td>0.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. Effect of density ratio 41.3 µm (\rho_{film}/\rho_{water} = 0.5, 1, 1.5)</td>
<td>53</td>
<td>0</td>
<td>90° (normal)</td>
<td>5 µm</td>
<td>0.12</td>
<td>1 590</td>
<td>1 680</td>
</tr>
</tbody>
</table>
FIG. 7. Convergence of crown radius at $T = 1$. The effective grid resolutions are 40, 52, 62, and 74 cells per diameter. The percentages show that the dimensionless crown radius increases when increasing grid resolution.

1. Validation case 1

We first validate the code with experiment of normal impact on thick liquid film. The experiment was conducted by Cossali et al. The water drop diameter was 3.82 mm, the terminal impact velocity was 3.94 m/s, the film thickness was 2.57 mm, and the resulting $We$ was 842 and $Re$ was 15 366. Our simulation has the same initial conditions as the experimental setup. A grid convergence study is performed using different effective grid resolutions defined by the number of cells per initial diameter of the drop. As shown in Fig. 7, the effective grid resolutions of 40, 52, 62, and 74 cells per diameter (cpd) are used to measure the convergence of crown radius ($r_c$) at $T = 1$. $T$ is the dimensionless time, which is defined as $T = \frac{V_0}{D_0}$. The crown radius only increases by 1.7% when the grid resolution is increased from 62 cpd to 74 cpd, but the total central processing unit (CPU) time to finish the simulation to $T = 4$ almost doubled. To keep the computational cost relatively low with reasonable simulation accuracy, the grid resolution of 62 cpd is adopted in this case. Similar grid convergence study is carried out for all simulation cases in this paper.

Figure 8 compares our simulation result with the experiment. Only a quarter of the domain is simulated and the simulation result is mirrored to show the full drop impact. As shown in Fig. 8, our simulation matches the experimental result to capture the jets ejected from the crown and the formation of secondary drops. The experiment shows ripples inside the crown, which is possibly caused by the shape oscillation of the drop. Because no such perturbation exists in the simulation, the ripple effect is not observed.

To better illustrate the drop impact phenomenon, we use different materials to represent the liquid drop and liquid film even though both have the same properties of water. Throughout this paper, we use the following color code to represent different materials in three-dimensional simulations: white for air, gray for solid substrate, green for liquid film, and blue for liquid drop. Simulating a drop and liquid film in different materials allows for the mass originating in the drop and the liquid film to be tracked through the impact process. Our numerical simulation shows that the drop mainly embeds inside the crown and most of the liquid in the crown is from the liquid film. Rim of the crown and secondary drops have a mixed color of green and blue, indicating that they are mixtures from the drop and the liquid film.

We validate our code quantitatively against experimental measurement by Cossali et al. of a 3.82-mm-diameter water drop impact onto a 2.57-mm-thick water film with the impact velocity of 3.51 m/s ($We = 667$, $Re = 13 676$, and $\delta = 0.67$). Cossali et al. measured the time evolution of the outer diameter of the formed crown. As shown in Fig. 8(a), the crown diameter is measured from the base of the ejected jets. Our simulated time evolution of the crown diameter is compared with the...
experimental results in Fig. 9. The rim of the crown moves relatively fast outward at the beginning at a speed of about 3 m/s and then it slows down to about 0.5 m/s at the end of the simulation. The simulation shows good agreement with the experimental result.

In the code validation with low-speed drop impact on thick liquid film, our simulation tool shows good agreement with experiments qualitatively and quantitatively. The crown shape is similar to experimental observation and the time evolutions of crown are in agreement.

2. Validation case 2

In the second validation case, we compare the simulation result with experiment of low-speed impact on a thin liquid film. The experiment conducted by Zhang et al. studied 2-mm-diameter silicon oil impact on the same liquid film with $We = 324$, $Re = 2191$, and $\delta = 0.2$. The simulation of 2-mm-diameter silicon oil drop impact on the same 0.4-mm-thick liquid film with the impact velocity of 1.83 m/s is conducted, which leads to the same $We$ and $Re$. The physical properties of the liquid in the simulation are the same as in the experiment. We use two-dimensional axisymmetric coordinates in the simulation because secondary drop ejection or crown breakup is not observed in the experiment. The effective grid resolution is 780 cpd. Figure 10(a) shows the simulation result with the drop and the liquid film distinguished in different colors. The air bubbles trapped under the drop are shown in blue, and a similar air entrainment phenomenon is observed in experiments. Figure 10(b) compares the liquid surface profiles between the experimental image and numerical simulation at $t = 0.335 \mu s$ and $0.331 \mu s$, respectively. The comparison shows that our simulation is in good agreement with the experiment.

3. Validation case 3

The third case validates the code for high-speed normal impact of a drop on thin liquid film. The water drop has a diameter of 41.3-μm and two thin film thicknesses, 1 and 5 μm ($\delta = 0.024$ and 0.12), are used. The normal impact velocity is 53 m/s. Only a quarter of the drop is simulated due to the symmetry of the problem. A grid convergence study is first carried out and the grid resolution of 46 cells per drop diameter shows good convergence of crown radius.

The simulation results are shown in Fig. 11. In both cases, the drop completely repels the thin film from the substrate after contact. The drop liquid is pushed radially outward, which is resisted by the adjacent film liquid at rest. Following the direction of least resistance, a thin sheet of liquid is propelled upward to form a crown from the neck region connecting the two liquids. When the drop keeps pushing the thin crown outward, the crown is radially stretched and eventually breaks up into secondary drops. Based on their experiments, Rioboo et al. observed that when the dimensionless film thickness, $\delta$, is smaller than 0.03, the drop splashes without crown formation. Nevertheless, Fig. 11 clearly demonstrates the crown formation for $\delta = 0.024$. One possible reason is that the experiments of Rioboo et al. were conducted at low velocities ($V_0 < 3.14$ m/s), and their observation may not be valid for high-speed drop impact. We hypothesize that the high-speed impingement creates a high speed thinner lamella moving outward, leading to higher resistance and upward motion of the lamella.
A close comparison of the two cases in Fig. 11 reveals some distinct features. First, the horn-shaped jet formed from the neck region appears earlier for the thinner film case. Second, the thinner film case has earlier crown breakup and more secondary drops. Third, the thinner film case has thinner crown that moves faster both outward and upward. The crown motions can be quantitatively measured, as shown in Fig. 12, and the time evolutions of crown radius and height are plotted. The crown radius, \( r_c \), and crown height, \( h_c \), are measured from the base of the ejected jets, as illustrated in Fig. 8(a). Comparisons are also made with the theoretical prediction of crown radius change with time. The theoretical prediction is derived by Yarin and Weiss\(^1\) and is given by

\[
\frac{r_c}{D} = \left( \frac{2}{3} \right)^{1/4} \frac{V_0^{1/2}}{D^{1/4} h_0^{1/4}} (t - t_0)^{1/2},
\]

where \( t_0 \) is the shifting time and \( t_0 = 0.02 \mu s \). The time evolutions of crown radius from our simulations qualitatively agree with the theoretical predictions. It is also clear from Fig. 12 that the thinner film case has larger crown radius and higher crown than the thicker film case. For the thinner film case, the crown moves faster in both outward and upward directions because thinner crown has less inertia and hence more tendency to be pushed by a drop.

B. Effect of impact angle

To understand the effect of impact angle on drop impact on liquid film, we maintain the same normal impact velocity but vary the tangential velocity to modify the impact angle. Our simulations reveal that smaller impact angle leads to smaller
lamella on the downstream side and suppression oflamella development on the upstream side.

In all simulation cases, the water drop diameter is 41.3 \( \mu \text{m} \), the film thickness is 5 \( \mu \text{m} \), and the normal component of impact velocity is 53 m/s. Two impact angles are first studied: 30° and 60°. Because the problem is plane-symmetric, only one half of the drop is simulated for the three-dimensional numerical model. Snapshots of impact at different time instants are shown in Fig. 13. The two cases share similarities. In both cases, a vertical liquid lamella is formed on the downstream side. The lamella originates near the intersection between the drop and the liquid film after the impact. Then the lamella moves obliquely upward. The rim of the lamella becomes unstable and breaks up into secondary drops, similar to the normal impact cases shown in Fig. 11. The green color of the lamella indicates that the liquid is mainly from the pre-existing liquid film.

However, the differences in flow patterns and splashing behaviors are also obvious even though their normal impact velocities are the same. As shown in the 60° impact angle case (Fig. 13 left), crown is formed almost immediately after the impact. Then the drop continues to push the crown lamella moving outward and breaks the crown into secondary drops. During the crown development, the blue color only appears on the inside of the crown lamella, indicating that the drop liquid slides on top of the lamella. However, in the 30° impact angle case (Fig. 13 right), completely different behaviors downstream of the drop are shown. No crown is formed on the left of the drop at time instants of 0.60 \( \mu \text{s} \) or 0.81 \( \mu \text{s} \). Our simulation shows that the formation of a crown is not encouraged as a result of increasing tangential velocity. Instead, the drop rolls over onto the film liquid at time instants of 0.81 \( \mu \text{s} \) and 1.25 \( \mu \text{s} \), as shown in the zoomed-in view of Fig. 14. The rolled-up liquid then re-impinges on the quiescent film in front to form splashing. Instead of breaking up into small secondary drops, long liquid fingers are formed by the re-impingement. Both cases also show that the solid surface emerges at the impact site at the later impact stage, as seen at 6.79 \( \mu \text{s} \), and the solid surface at the impact site is almost dry with small amount of drop liquid remains on the solid surface.

As the tangential velocity increases, a larger portion of the drop moves toward the upstream side, leaving less liquid drop downstream. Thus, as shown in Fig. 15, the crown radius, \( r_c \), and crown height, \( h_c \), on the downstream side are both smaller for the 30° impact angle case than for the 60° impact angle case. The crown radius, \( r_c \), and crown height, \( h_c \), are measured from the base of the ejected jets, as illustrated in Fig. 8(a).

To highlight some prominent flow features with higher grid resolution, we simulate the two cases using twodimensional numerical models. Figure 16 compares the simulation results. The 30° impact angle case has a higher tangential velocity and the velocity difference across the interface incurs strong Kelvin-Helmholtz instability, which is clearly seen at
the interface between the drop and film. Due to perturbation and tangential motion of the upper liquid, the interface evolves into an unstable vortex sheet that rolls up into spirals. Another difference between the 30° and 60° impact angle cases is the liquid volume portions in the expanding lamella at the upstream side. The 30° impact angle case shows that the liquid from drop suppresses the upward movement of liquid film, while the 60° impact angle case shows that the two liquids move outward together. More liquid volume from the film is shown in the lamella in the 60° impact angle case than in the 30° impact angle case.

Splashing can be completely suppressed on the upstream side with the increase of tangential impact velocity. As shown in Fig. 17, at the 15° impact angle (tangential impact velocity of 197.8 m/s), no lamella is developed on the upstream side.

C. Effect of film thickness

To study the effect of film thickness on oblique impact, we simulate water films of 1 ȝm, 5 ȝm, and 20 ȝm. In all cases, the drop has a diameter of 41.3-ȝm, impact angle of 45°, and normal impact velocity of 53 m/s. As shown in Fig. 18, splashes are observed in all cases. On the downstream side, the vertical lamella is thin and breaks up almost immediately after its formation for the 1 ȝm case, while the lamella is much thicker and lamella breakup is significantly delayed for the 20 ȝm case. Because the formed lamella on the downstream side is mainly from liquid film, thinner film leads to thinner lamella, and thinner lamella is less stable and breakup happens sooner. On the upstream side, increasing film thickness also causes thicker lamella, and the upward motion of the thicker lamella is more prominent. Because a thicker quiescent film has more inertia and hence greater tendency to resist a change in motion, the lamella’s upward motion becomes more prominent due to less resistance from air.

D. Effect of film-drop density ratio

We vary the liquid film density while keeping the drop density constant to study the effect of density ratio of liquid film and drop. The simulations are performed in two-dimensional axisymmetric coordinate systems. Figure 19 shows numerical results of 41.3-ȝm-diameter water drops impact onto 5-ȝm-thick liquid films of different densities. The impacts are normal and the impact velocity is 53 m/s for all cases.

Figure 19 clearly shows that lowering the film density leads to early splashing: splashing occurs first at 0.07 ȝs for the lowest film density case (ρ_film = 0.5 ρ_water), while splashing occurs last at 0.26 ȝs for the highest film density case (ρ_film = 1.5 ρ_water). For the ρ_film = 0.5 ρ_water case, the crown formation and breakup are also observed at the early stage of the impact similar to the ρ_film = ρ_water case. Then, from t = 0.37 ȝs, the drop starts to spread beneath the film and a Kelvin-Helmholtz vortex is developed. Unlike the outward and upward motions of the expanding crown in the ρ_film = ρ_water case, the crown collapses inward and splashing is suppressed at the later stage of the impact when ρ_film = 0.5 ρ_water. For the ρ_film = 1.5 ρ_water case, splashing is also attenuated with lower crown height and smaller crown radius because higher film density indicates that the crown has more inertia and less tendency to be pushed radially.

Figure 19 shows that at t = 0.07 ȝs in all cases, a horn-shaped jet is produced immediately after impact in the neck region where the drop contacts the liquid film. Although the jet shapes are similar and the crown radius r_c = 0.42 for all cases, obvious differences can be seen. For the ρ_film = 0.5 ρ_water case, the jet is from liquid film and already breaks up at 0.07 ȝs; for the ρ_film = ρ_water case, liquid from drop starts to move outward above the jet from liquid film; for the ρ_film = 1.5 ρ_water case, liquid from drop moves above and ahead of the jet from liquid film. At t = 0.18 ȝs, the crown tips of the ρ_film = 0.5 ρ_water case and the ρ_film = ρ_water case break up into secondary drops, while the ρ_film = 1.5 ρ_water case keep together.
the $\rho_{\text{film}} = 1.5\rho_{\text{water}}$ case shows that the liquid from drop moves over the jet from liquid film and re-impacts the film to create a crater in front of the crown. At $t = 0.26$ µs, the remaining liquid film beneath the drop of the $\rho_{\text{film}} = 0.5\rho_{\text{water}}$ case is much thinner compared with the other cases.

**IV. CONCLUSIONS**

We numerically investigated the dynamics of high-speed oblique impact of a liquid drop on a thin liquid layer. The Navier-Stokes equations are solved using the projection method on Cartesian grids and the moment-of-fluid method is used to construct the interfaces. We validated the code first by comparing the numerical results with the experimental results and the theoretical predictions, and good agreement was achieved qualitatively and quantitatively. We studied the effect of the impact angle by changing the tangential component of drop impact velocity while maintaining a constant normal component of impact velocity. Then we studied the effect of film thickness for oblique impacts with fixed impact angles. Finally, we studied the effect of film-drop density ratio by changing liquid film density.

The following important conclusions can be made from the present study: (i) The tangential impact velocity of the incident drop affects the outcome of oblique drop impact. On the side behind the advancing drop, higher tangential velocity leads to lower lamella height and smaller lamella radius. On the side in front of the advancing drop, higher tangential velocity induced Kelvin-Helmholtz vortices development at the drop and liquid film interface and tends to suppress the evolution of lamella. (ii) Liquid film thickness affects the outcome of oblique drop impact. Thinner liquid film leads to thinner expanding crown and earlier crown breakup. (iii) Film density affects the outcome of oblique drop impact. Lower film density can prompt earlier splash but drop tends to move beneath the liquid film at the later stage of impact.

Our study revealed that the tangential component of impact velocity is not negligible for oblique drop impingement, especially for high-speed impact. The tangential motion introduces large shear stress on the side in front of the advancing drop, which may prompt or suppress splash depending on the impact angle. The effect of film-drop density ratio is not observed in existing studies and certainly needs further investigation both experimentally and numerically.


